Proceedings of the
1st International Conference on Environmental Health, Socioeconomic and Technology 2022
Lamongan, Indonesia, August 10-11 2022
Vol. 1, No. 1 (https://conference.unisla.ac.id/index.php/icehst/)

MODELLING OF DENGUE VIRUS TRANSMISSION WITH OPTIMAL CONTROL

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ABSTRACT

An infected female mosquito bite is how the virus that causes dengue fever is spread to humans. In this research, a mathematical model that includes vector control and human susceptibility awareness is suggested to describe the transmission of the two strains of Dengue virus between humans and mosquitoes. This study aims to establish and analyze a mathematical model of dengue fever with the application of optimal control. The results of this study are expected to be able to provide information to the government, as well as material for further research. The method used to solve the above problem is to use the Pontryagin maximum principle method, which is then solved by the fourth-order runge kutta numerical method. The simulations carried out showed that the population of susceptible and infected human individuals decreased with optimal control, while the population of recovered individuals increased after optimal control was given. In the mosquito population, after being given optimal control, the mosquitoes capable of being infected (susceptible) and the mosquitoes infected with the dengue virus (infected) decreased compared to before the optimal control was given. This shows that the optimal control works well on the mathematical model of dengue fever. Theoretical results and numerical simulations indicate that measures to increase awareness of the self-protection of infected and susceptible humans should be taken and mosquito control measures are needed to prevent transmission of Dengue virus from mosquitoes to humans.

KEYWORDS: *Dengue Virus, Mathematical Model, Optimal Control*

1 INTRODUCTION

Aedes aegypti and Aedes albopictus, which are known as the primary vectors of dengue (L. Esteva & Vargas, 2003; Lourdes Esteva & Vargas, 1999), are two species of mosquitoes that can carry the virus that causes dengue fever and bite humans. Because of the significant morbidity and mortality associated with dengue fever, which occurs in the majority of tropical, subtropical, and temperate countries, the disease has recently gained international attention as a public health concern. High fever, a frontal headache, discomfort behind the eyes, joint problems, nausea, vomiting, and other symptoms are some of the signs and symptoms of this illness (Derouich & Boutayeb, 2006). According to a recent statistics, there are 390 million Dengue outbreaks each year (95% credible interval: 284-528 million), of which 96 million (range: 67–136 million) show clinical symptoms (of any illness severity) (Bhatt et al., 2013).

How to prevent and manage the spread of this disease has been a hot problem from many perspectives, including medical scientists and mathematicians, as more than one-third of the world's population lives in places at risk for infection with the Dengue virus (Centers for Disease Control and Prevention). To understand the dynamic behaviors of dengue transmission, numerous mathematical models have been put forth thus far. As an overview, Esteva and Vargas (Lourdes Esteva & Vargas, 1999) proposed a SIR model for the transmission of dengue fever with variable human population size. They discovered three threshold parameters that regulate the existence of the behaviors of the total number of infected humans, the growth of the human population size, and the endemic proportion equilibrium. Considered the effect of vector-control techniques on the occurrence of the Dengue virus in humans in Amaku et al. (Amaku et al., 2014). A deterministic model of the dynamics of dengue fever transmission proposed by Garba et al. (Garba et al., 2008) showed that the disease-free equilibrium exists and is locally asymptotically stable when the fundamental reproduction number is smaller than unity. In addition, authors covered the reverse bifurcation phenomenon. Blayneh et al. (Blayneh et al., 2009), Cai and Li (Cai & Li, 2010), Cai et al. (Cai et al., 2017), Esteva and Vargas (Lourdes Esteva & Vargas, 1998, 2000), Rodrigues et al. (Rodrigues et al., 2010), and the research in this area is still ongoing, all provide additional examples.

This work examines the creation of a mathematical model of dengue disease with optimal control based on the justification provided. An intervention strategy to reduce mosquito populations by spraying adult mosquitoes to kill them, using barrier techniques like curtains, avoiding places where mosquitoes congregate, and wearing long sleeved clothing to increase the level of protection of vulnerable humans is suggested as the best control in this study. With the use of optimum control, this work intends to identify and examine the mathematical model of dengue hemorrhagic fever. Whereas the goal of this optimal control is to lower both the number of infected people and the number of Aedes aegypti mosquitoes. The findings of this study are anticipated to offer the government with knowledge and inspiration for additional research.

The following sections serve as the structure for this article. Section 2 explains dynamic model of dengue virus. Section 3 discusses the mathematical representation of dengue disease using optimal control and its properties. Then the numerical simulation and discussion explain in Section 4. The final section includes some conclusion of this research.

2 DENGUE EPIDEMIC MODEL

The underlying presumptions were as follows, one of the serotypes of the dengue virus can be transmitted between the host and the vector in a mathematical model called SIR random. The model is a form of SIR model since it is based on the disease's eliminated traits, infection, and susceptibility. The human population (N_h) and the vector population are the two separate sorts of populations on which the model is developed (N_v). The human population N_h is further broken down into three groups: those who may have the dengue virus but are still susceptible to infection X_h (also known as susceptible), those who have the dengue fever Y_h (also known as infected), and those who have recovered from the illness Rh (removed). Similar to this, mosquitoes that are susceptible to infection (X_v) and mosquitoes that are infected with the dengue virus (N_v) make up the two categories that make up the vector population of mosquitoes (infected, Y_v). The dengue vaccine is now thought to offer everlasting immunity. Because of this, we hypothesized that following a successful vaccine, the vulnerable population would migrate to the recovered population. The proposed model assumes non-negative real numbers for all of its parameters and state variables. For the purpose of the noise effect, the functions $B_1(t), B_2(t), B_3(t)$, whereas $1, 2, 3 > 0$ are taken to be the equivalent intensities of the white noise. The fundamental axiom $B_1(0) = B_2(0) = B_3(0) = 0$. The person who recovers from the illness will always be immune to it.

The mathematical model SIR random can replicate the spread of serotypes, one of the dengue viruses, from the vector to the host. The model is actually a form of SIR model because it is based on susceptibility, infection, and eliminated aspects of the disease. The human population (N_h) and the vector population are the two separate sorts of populations on which the model is built (N_v). The human population N_h is further broken down into three groups: those who may already have the dengue virus but are still susceptible to infection X_h (also known as susceptible), those who have the dengue fever Y_h (infected), and those who have recovered from the illness R_h (removed) (Din et al., 2021). In this case, the governing system of equations can be written as follows:

$$\frac{dX_h(t)}{dt} = \Lambda - \frac{\beta_1 X_h(t) Y_h(t) + \beta_2 X_h(t) Y_v(t)}{N_h} - \mu_0 X_h(t) \tag{1}$$

$$\frac{dY_h(t)}{dt} = \frac{\beta_1 X_h(t) Y_h(t) + \beta_2 X_h(t) Y_v(t)}{N_h} - (\mu_0 + \mu_2 + \gamma_1) Y_h(t) \tag{2}$$

$$\frac{dR_h(t)}{dt} = \gamma_1 Y_h(t) - \mu_0 R_h(t) \tag{3}$$

$$\frac{dX_v(t)}{dt} = \lambda - \frac{\beta_3 X_v(t) Y_h(t)}{N_h} - \mu_1 X_v(t) \tag{4}$$

$$\frac{dY_v(t)}{dt} = \frac{\beta_3 X_v(t) Y_h(t)}{N_h} - \mu_1 Y_v(t) \tag{5}$$

3 OPTIMAL CONTROL PROBLEM

The best strategies for preventing the spread of the dengue virus are developed using optimal control techniques. In this section, we provide an optimal problem for model (1)-(5) to determine a good balance between the least number of total mosquitoes, and campaign costs, the infected individuals, also the susceptible individuals. The levels of disease knowledge in the host population and efficient vector control are seen as a control variable to lessen or even abolish the illness.

The sensitivity analysis performed is used in this section to modify the social hierarchy-structured model (1)-(5) to add the next two intervention options.

i. $u_1(t)$: This control variable is an intervention strategy to reduce the mosquito population by spraying adult mosquitoes to kill them.

ii. $u_2(t)$: The suggested method involves employing barrier techniques like curtains and avoiding spots where mosquitoes are swarming and long sleeved garments to increase the degree of susceptible humans' self-protection.

The mathematical model of the spread of the dengue virus by providing optimal control is given by the following system:

$$\frac{dX_h(t)}{dt} = \Lambda - \frac{\beta_1 X_h(t)Y_h(t)+\beta_2 X_h(t)Y_v(t)}{N_h} - \mu_0 X_h(t) - u_1 X_h(t) \tag{6}$$

$$\frac{dY_h(t)}{dt} = \frac{\beta_1 X_h(t)Y_h(t)+\beta_2 X_h(t)Y_v(t)}{N_h} - (\mu_0 + \mu_2 + \gamma_1)Y_h(t) \tag{7}$$

$$\frac{dR_h(t)}{dt} = \gamma_1 Y_h(t) - \mu_0 R_h(t) + u_1 X_h(t) \tag{8}$$

$$\frac{dX_v(t)}{dt} = \lambda - \frac{\beta_3 X_v(t)Y_h(t)}{N_h} - \mu_1 X_v(t) - u_2 X_v(t) \tag{9}$$

$$\frac{dY_v(t)}{dt} = \frac{\beta_3 X_v(t)Y_h(t)}{N_h} - \mu_1 Y_v(t) - u_2 X_v(t) \tag{10}$$

In order to decrease or eradicate the numbers of both infected populations and mosquitoes, the time-dependent control variables $u_i(t),\ i = 1, 2$, are included. The cost functional listed below is utilized as a result.

$$\min J\,(u_1, u_2) = \int_0^{t_{end}} \left(A_1 X_h + A_2 Y_h + \frac{A_3}{2}u_1^2 + \frac{A_4}{2}u_2^2\right) dt \tag{11}$$

which A_1, A_2, A_3, A_4 are the balancing and positive weight constants for reducing the function. The nonlinearity of the costs associated with each intervention strategy required to reduce the objective functional deficit is described by each of the quadratic equations u_i^2 (Ademosu et al., 2021; Adepoju & Olaniyi, 2021; Asamoah et al., 2020; Goswami & Shanmukha, 2020; Khan et al., 2021). Tf indicates that the intervention measures should be implemented for the last time. In order to solve the minimization issue of the type, one only needs to find an optimal control $u = (u_i),\ \ i = 1, 2$. Utilizing Pontryagin's maximal principle (Ilmayasinta et al., 2022; Ilmayasinta & Purnawan, 2021; Mardlijah et al., 2018, 2019; Pontryagin, 1986), the minimization issue (11) is solved using optimal control theory.

3.1 Optimal control characterization

It's critical to demonstrate the presence of the optimal control quadruple u that found the solution to the minimization problem (11). It is given beneath.

$$H = A_1 X_h + A_2 Y_h + \frac{A_3}{2}u_1^2 + \frac{A_4}{2}u_2^2 + \xi_1 \left(\Lambda - \frac{\beta_1 X_h(t)Y_h(t)+\beta_2 X_h(t)Y_v(t)}{N_h} - \mu_0 X_h(t) - u_1 X_h(t)\right) + \xi_2 \left(\frac{\beta_1 X_h(t)Y_h(t)+\beta_2 X_h(t)Y_v(t)}{N_h} - (\mu_0 + \mu_2 + \gamma_1)Y_h(t)\right) + \xi_3(\gamma_1 Y_h(t) -$$

$$\mu_0 R_h(t) + u_1 X_h(t)) + \xi_4\left(\lambda - \frac{\beta_3 X_v(t) Y_h(t)}{N_h} - \mu_1 X_v(t) - u_2 X_v(t)\right) + \xi_5\left(\frac{\beta_3 X_v(t) Y_h(t)}{N_h} - \mu_1 Y_v(t) - u_2 X_v(t)\right) \tag{12}$$

where $\xi_1, \xi_2, \xi_3, \xi_4, \xi_5$ are, respectively, the adjoint variables linked to the state variables X_h, Y_h, R_h, X_v, Y_v. The next result illustrates both the presence of the adjoint variables and the control characterisation.

Theorem. There are adjoint variables $\xi_1, \xi_2, \xi_3, \xi_4, \xi_5$ that fulfill for adjoint system of certain form, provided an optimal control (u_1^*, u_2^*) that minimizes the objective functional (11).

$$\dot{\xi_1} = -\left(1 - \xi_1(\beta_1 Y_h(t) + \beta_2 Y_v(t) + \mu + u_1) + \xi_2\left(\beta_1 Y_h(t) + \beta_2 Y_v(t)\right) + \xi_3 u_1\right) \tag{13}$$

$$\dot{\xi_2} = -\left(1 - \xi_1\left(\beta_1 X_h(t)\right) + \xi_2\left(\beta_1 X_h(t) - (\mu_0 + \mu_2 + \gamma_1)\right) + \xi_3\gamma_1 + \xi_4\left(\beta_3 X_v(t)\right) + \xi_5(\beta_3 X_v(t))\right) \tag{14}$$

$$\dot{\xi_3} = -(-\xi_3\mu_0) \tag{15}$$

$$\dot{\xi_4} = -\left(\left(-\xi_4(\beta_3 Y_h(t) + \mu_1 + u_2)\right) + \xi_5(\beta_3 Y_h(t))\right) \tag{16}$$

$$\dot{\xi_5} = -\left(\left(-\xi_1(\beta_2 X_h(t))\right) + \xi_2(\beta_2 X_h(t)) - \xi_5(\mu_1 + u_2)\right) \tag{17}$$

considering final-time constraints or transversality

$$\xi_k(t_{end}) = 0, k = X_h, Y_h, R_h, X_v, Y_v, \tag{18}$$

and characterizations for optimal control

$$u_1^* = min\left\{1, max\left\{0, \frac{\left((\xi_1 X_h(t)) - \xi_3 X_h(t)\right)}{A_3}\right\}\right\} \tag{19}$$

$$u_2^* = min\left\{1, max\left\{0, \frac{\left((\xi_4 X_v(t)) - \xi_5 Y_v(t)\right)}{A_4}\right\}\right\} \tag{20}$$

Proof. The adjoint system is generated by partial differentiating the Hamiltonian (12) with respect to each of X_h, Y_h, R_h, X_v, Y_v (13)-(17). Additionally, the two optimal controls are characterized by solving as follows:

$$\frac{\partial H}{\partial u_1} = 0; \tag{21}$$

$$u_1 = \frac{\left((\xi_1 X_h(t)) - \xi_3 X_h(t)\right)}{A_3} \tag{22}$$

$$\frac{\partial H}{\partial u_2} = 0; \tag{23}$$

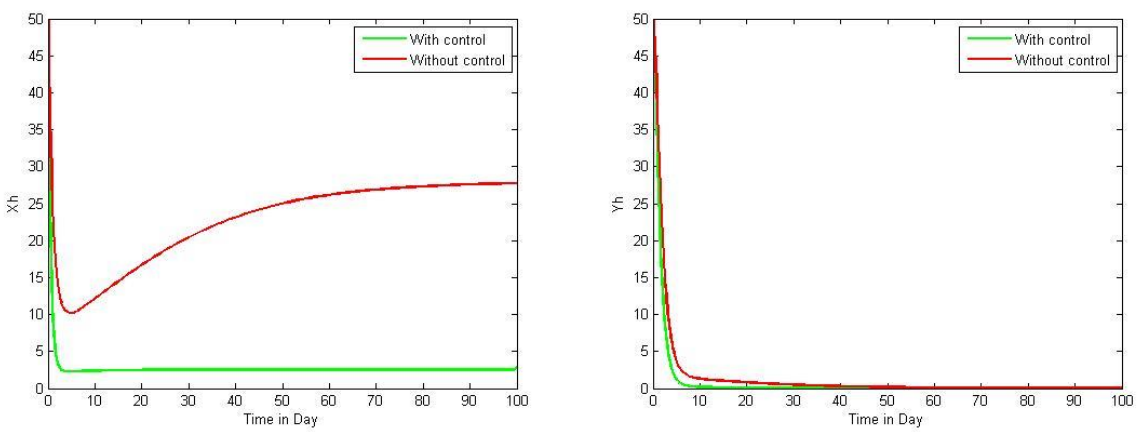
$$u_2 = \frac{\left((\xi_4 X_v(t)) - \xi_5 Y_v(t)\right)}{A_4}. \blacksquare \tag{24}$$

4 NUMERICAL SIMULATION AND DISCUSSION

The outcomes of simulations on the dengue disease mathematical model with and without optimal control are shown in Figure 1 and the values of the parameters used are presented in table 1.

Table 1 Description and Value of the Parameters

Notation	Parameters Description	Value
γ_1	Infected population's rate of recovery	0.8
β_3	The proportion of infected mosquitoes that bite the general public without being infected	0.02
β_2	Rate of exposure to non-infected mosquitoes by infected people	0.002
β_1	The frequency of contact between infected and uninfected people	0.015
μ_2	The disease's mortality rate	0.02
μ_1	Rate of deaths caused by mosquitoes	0.1
μ_0	Death rates among people	0.1
λ	Birth rate of mosquitoes	2
Λ	Rate of births among people	2.8



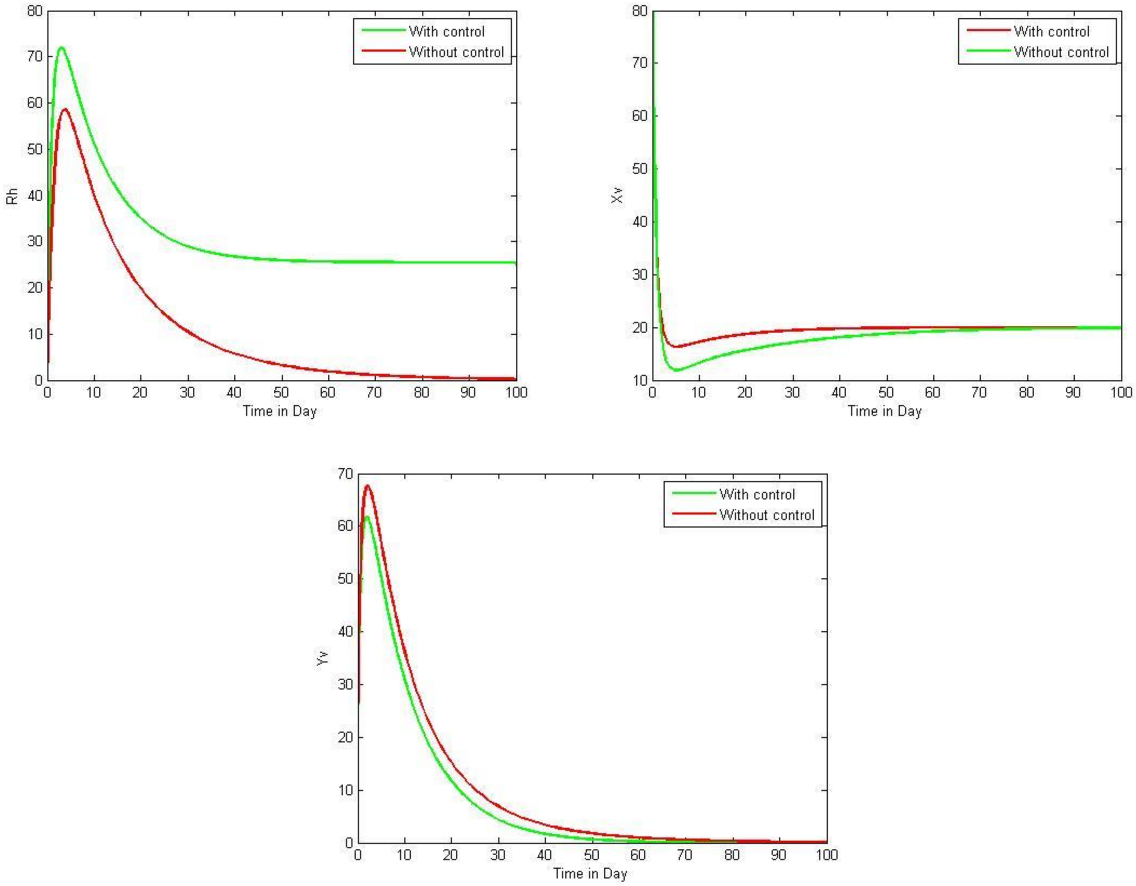


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Figure 1: Simulations of $(X_h, Y_h, R_h, X_v, Y_v)$ with and without optimal control

Prior to receiving optimal control at the start of the simulation, state X_h shown a drop, but after day 5 it underwent a large increase up until the end of the observations made. The population suffered a large decline on the first and second days after receiving optimal control; however, after the decrease, the population remained stable until day 100. State Y_h, before and after being provided optimal control at the start of the simulation, suffered a large decline, which on the days then also experienced a minor decrease but headed to 0 until the end of the observation. The deterioration accelerated slightly more quickly after receiving control than it had earlier. When given optimal control, the behavior of the obtained graph in the population of recovered individuals remained constant. However, after receiving optimal management, the increase was more noticeable. Before being given control, it did rise at the start of the observation, but around day 10, it started to fall until it reached zero, showing that there was no population of recovered persons at the end of the experiment. Following the application of optimal control, it climbed following the first observation. Subsequently, it also declined, but this decline was not substantial and did not reach 0. In state X_v, after receiving optimal control, the vector population, or mosquito population, initially showed a more pronounced decline at the beginning of the observation, albeit on the following days there was a modest increase that remained steady until the end of the observation. Prior to the tenth day, the last state or Y_v vector population grew at the start of the observation before decreasing. The graph reaches zero more quickly than it would have otherwise when control is given because the drop is more noticeable when optimal control is applied.

5 CONCLUSION

In this research, a methodical analysis of the dynamics of a two-strain Dengue fever model with vector control and awareness of sensitive humans is presented. A variety of anti-Dengue preventive measures can be evaluated using the model, which incorporates key aspects of Dengue fever transmission. The number of infected people can be lowered, according to numerical simulations, by using awareness-raising tactics for protecting vulnerable people personally and controlling vectors. The Figure 1 illustrates how, following the application of the control, there were fewer susceptible people, infected people, and mosquito populations in the two populations that were taken into consideration. There was a noticeable increase in the population of patients who made a full recovery after receiving control. So taking the two strategies, it was concluded that they were able to prevent transmission of dengue fever in humans and mosquitoes as host and vector control of the mathematical model of dengue fever, and the control provided could work well on the mathematical model of dengue fever. Based on the mechanism of transmission of the dengue virus, reducing the access of mosquitoes to the dengue virus is very important to prevent transmission of the disease itself.

6 ACKNOWLEDGEMENTS

This research was funded by the Research Funds of Universitas Islam Lamongan.

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