

## STUDY ON THE PROBLEMS OF PARTICLE DYNAMICS AND RIGID BODIES (MECHANICAL ENGINEERING FIELD) AND ELECTRICAL CIRCUITS (ELECTRICAL ENGINEERING)

Ulul Ilmi<sup>1</sup>, Heri Purnawan<sup>2</sup>

<sup>1,2</sup> Universitas Islam Lamongan, Jl. Veteran No 53 A, Lamongan, Indonesia

[ululilmi78@yahoo.co.id](mailto:ululilmi78@yahoo.co.id), [heripurnawan@unisla.ac.id](mailto:heripurnawan@unisla.ac.id)

### ABSTRACT

*In this study, the problem of particle dynamics and rigid body (mechanical engineering) and electrical circuits (electrical engineering) to be solved is a system of linear equations consisting of several variables from the problem of particle dynamics and rigid body (mechanical engineering) and electrical circuits (electrical engineering). For the problem of particle dynamics and rigid body (mechanical engineering) contains variable of acceleration, mass and force of the object. While the problem of the electrical circuit consisting of a variable electrical resistance, one voltage source and the other voltage source are grounded. These two problems have the following general form:*

$$p_{11}x_1 + p_{12}x_2 + \dots + p_{1n}x_n = k_1$$

$$p_{21}x_1 + p_{22}x_2 + \dots + p_{2n}x_n = k_2$$

.....

$$p_{n1}x_1 + p_{n2}x_2 + \dots + p_{nn}x_n = k_n$$

*Where each  $p$  is a constant coefficient,  $k$  is a constant, and  $n$  is the number of equations. In this study, the matrix that will be generated from the problem of an electrical circuit with dimensions of  $10 \times 10$  is needed and the matrix of the problem of particle dynamics and rigid body has size of  $3 \times 3$ , so that the Matlab program is needed to obtain more accurate, easier and faster calculation results. To solve the problem of particle dynamics and rigid bodies (mechanical engineering) and electrical circuits (electrical engineering) the following steps are used: (1) Determine the problem, (2) Formulate the problem in the form of a system of linear equations, (3) Change the form of the system of equations linear into the matrix form  $[A][X] = [C]$ , (4) With the matlab program, the inverse of the matrix  $[X]$  is calculated by the formula  $[X] = [A]^{-1}[C]$ , (5) With this result can answer the research questions posed.*

**KEYWORDS:** system of linear equations, matlab, matrix  $AX = B$

# 1 INTRODUCTION

In this study, the problem discussed is the use of the MATLAB program to solve the problem of particle dynamics and rigid bodies (mechanical engineering) and electrical circuits (electrical engineering). The two problems to be solved are linear equations consisting of variable dynamics of particles and rigid bodies as well as some electrical resistance and a voltage source which has the following general form:

$$\begin{aligned} p_{11}x_1 + p_{12}x_2 + \dots + p_{1n}x_n &= k_1 \\ p_{21}x_1 + p_{22}x_2 + \dots + p_{2n}x_n &= k_2 \\ &\vdots \\ p_{n1}x_1 + p_{n2}x_2 + \dots + p_{nn}x_n &= k_n \end{aligned}$$

Where each  $p$  is a coefficient,  $k$  is a constant, and  $n$  is the number of equations. In this study, the matrix that will be generated from the problem of particle dynamics and rigid bodies (mechanical engineering) and electrical circuits (electrical engineering) has a maximum dimension of  $10 \times 10$  so that a matlab program is needed to obtain more accurate, easier and faster calculation results. .

## 1. Problem Formulation

Based on background description above, the research problems can be formulated as follows: How to construct a system of linear equations for solving problems of particle dynamics and rigid bodies (mechanical engineering) and electrical circuits (electrical engineering) ?

## 2. Problem Limitation

In this study, the problems discussed are limited to the problem of particle dynamics and rigid bodies (mechanical engineering) and electrical circuits (electronic engineering), namely a circuit consisting of several resistors connected in series and parallel where one end of the resistance is connected to a potential difference while the other end of the resistance is grounded/earthed..

## 3. Research Objectives

The purpose of this research is to find out how to construct linear equations, create programs for solving the problems of particle dynamics and rigid bodies (mechanical engineering) and electrical circuits (electrical engineering).

## 4. Theoretical Foundation

### 4.1 Particle Dynamics and Rigid Body (Mechanical Engineering)

Actually, the problem of particle dynamics and rigid bodies (mechanical engineering) is a problem that is still related to Newton's Laws. Newton's law itself is divided into three namely Newton's first law, Newton's second law and Newton's third law. To answer the problem of particle dynamics and rigid bodies, the following clauses are used in Newton's Second Law and Newton's

Third Law. Newton's second law contains the magnitude of the variable acceleration of an object depending on the magnitude of the object's force variable and inversely proportional to the value of the object's mass variable. While Newton's third law states that the magnitude of the variable value of an object's action force is proportional to the value of the object's reaction force variable, but has opposite directions.

#### **4.2 Electrical Circuits (Engineering electrical)**

The problem of electric circuits actually cannot be separated from Ohm's Law and Kirchoff's Law. Kirchoff's law can be divided into two, namely Kirchoff's law I and Kirchoff's II. Kirchoff's first law states:

The sum of the strong currents entering a branching point is the same as the sum of the strong currents leaving that branching point.

$$\sum i_{in} = \sum i_{out} \quad (1)$$

While Kirchoff's second law reads:

The algebraic sum of the changes in voltage around a closed circuit (loop), must be zero.

$$\sum V = 0$$

#### **5. System of Linear Equation**

The system of linear equations in this study represents the amount of electrical resistance  $R_{mn}$ , the amount of electric current  $I_{xy}$  and the electric potential difference  $V_{ab}$ . To construct a system of linear equations in electric circuits, Kirchoff's Law and Ohm's Law guidelines are used. Meanwhile, to develop a linear system of equations on the problem of particle dynamics and rigid bodies, Newton's Law is used. Kirchoff's law states that the algebraic sum of all currents entering a node must be equal to zero or

$$\sum i_k = 0 \quad (3)$$

Where all currents entering the node are considered positive. Ohm's law states that the current through a resistor is related to the change in the voltage and the value of the resistance. Mathematically written :

$$i_{ij} = \frac{V_i - V_j}{R_{ij}} \quad (4)$$

#### **6. Matrix**

After obtaining the form of the system of linear equations based on Kirchoff's Law and Ohm's Law, the next step is to change the form of the system of linear equations into matrix form. In this study, the matrix formed will have the same number of rows and the same number of columns. This is adjusted to the

conditions of the electrical circuit problems encountered. Because the matrix obtained from the system of linear equations in this study shows that the number of rows is the same as the number of columns, the inverse of the resulting matrix also has the form of a square matrix. The inverse of this matrix is the solution of the electric circuit in the form of a system of linear equations.

The solution produced by this electric circuit is in the form of a variable electric current and potential difference. If for example the matrix of the system of linear equations is matrix A, the resulting variable is said to be matrix X, while the constant value of the right hand side of the equal sign is said to be C, then mathematically it is written as:

$$[A] [X] = [C] \quad (5)$$

$$\text{or } [X] = [A]^{-1} [C] \quad (6)$$

with :

[A] = Variable matrix system of linear equations.

[X] = Calculated matrix variable (particle dynamics variable and rigid body and current and potential difference)

[C] = The constant value of the right side of the equals sign.

## 2 MATERIAL AND METHODS

Regarding the research method, this research will explain the block diagram of a linear equation system research for solving electrical circuits through computer programming as shown in Figure 1.

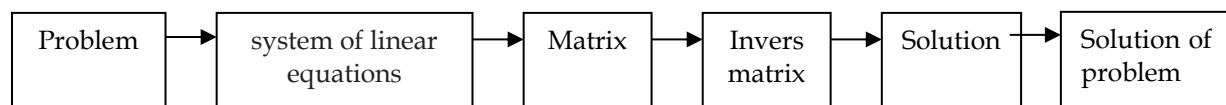


Figure 1. Research block diagram

First, determine the problem of the research. From this problem, a system of linear equations was developed. After creating a system of linear equations, the next step is to write it in matrix form. To solve this matrix, the inverse of the matrix is calculated with the help of matlab software. The inverse value that has been generated from the calculation of the matlab software is the value of the object's acceleration  $a$ , the object's mass, the object's force on the particle dynamics problem. In addition, there is also the resistance value  $R_{mn}$  and the potential difference  $V_{ab}$  between the two nodes in the electrical circuit.

In this study the flowchart used in solving this system of linear equations is as follows:

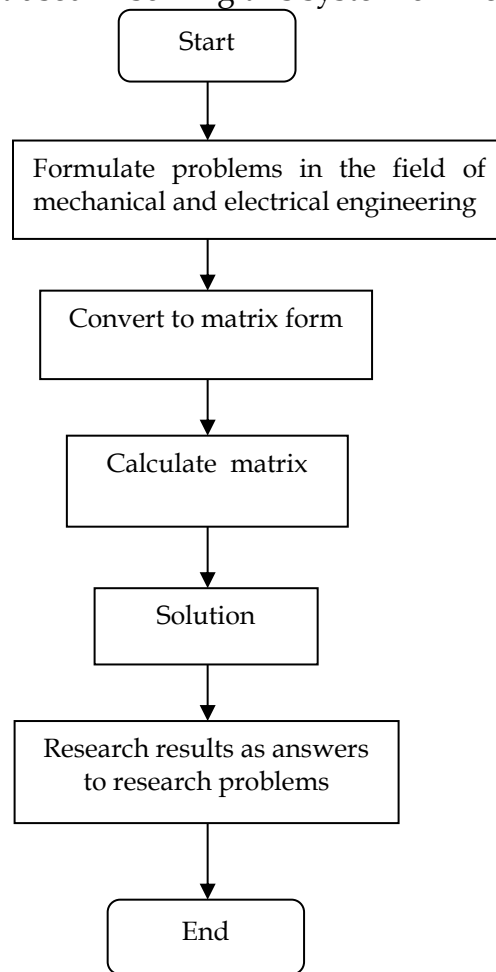


Figure 3. completion flowchart

The explanation of the flow diagram for solving a system of linear equations is:

1. Problems in mechanical and electrical engineering are formulated into a system of linear equations.
2. Write down the coefficients of the variable dynamics of the particle and rigid bodies as well as the current and the electric potential difference in the system of linear equations as a representation of the matrix  $[A]$ .
3. Write down the symbols of the current and electric potential difference variables in the system of linear equations as a representation of the matrix  $[X]$ . At the time of writing the program code (source code) in Matlab software, the symbols of the current and potential difference variables are represented by the matrix  $[X]$ .
4. Write down the value of the constant on the right side of the equals sign in the system of linear equations as a representation of the matrix  $[C]$ .

5. Calculate the matrix value  $[X]$  with the formula
6.  $[X] = [A]^{-1} [C]$ .
7. The value of the matrix calculation  $[X]$  is the value of the current and the electric potential difference as a solution for the electrical circuit.
8. Research results are answers to research problems

### 3 RESULT

The first problem encountered in research related to the field of mechanical engineering is the problem of particle dynamics and rigid bodies. For this reason, on this occasion the first problem will be discussed, namely the dynamics of particles and rigid bodies. Particle and rigid body dynamics play an important role in many other mechanical and engineering problems. This motion can be explained using Newton's laws. There are three Newton's Laws, which include Newton's first Law, Newton's second Law and Newton's third Law. Of the three Newton's laws, which will be used in this case are Newton's second law and Newton's third law. Why is that ? Because the magnitude that will be calculated regarding the object will move is the magnitude of the acceleration so that Newton's law used is Newton's second law which states that the magnitude of the acceleration of an object will be directly proportional to the force experienced by the object and inversely proportional to the mass of the object. In addition, in this case there is a law of action and reaction, so Newton's third law is used. Application of Newton's laws to a single particle yields two equations. But if some particles in a system affect others, then a large number of simultaneous equations will arise.

For example, suppose in this case the system shown in Figure 1 has three beams connected by a weightless wire at rest, on a smooth inclined plane that is 45 degrees to the horizontal. The coefficient of friction between the plane and a mass weighing 100 kg is 0.25 and with a mass weighing 50 and 20 kg is 0.375.

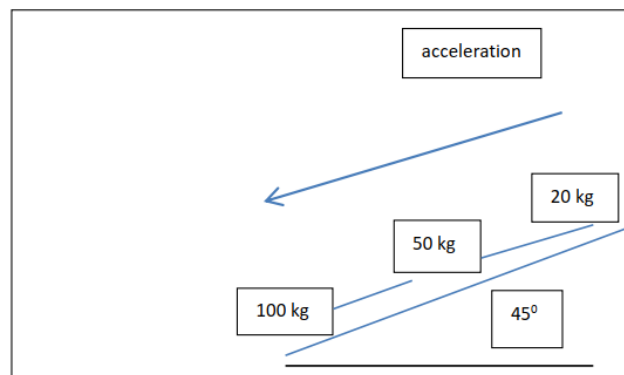


Figure 4. Three blocks connected by a wire rope without a load on an inclined plane

The free force diagram for the three blocks is given in Figure 4. The forces have units of Newton (kg per meter per second<sup>2</sup>), m is mass in kg, and a is acceleration in meters per second<sup>2</sup>. By adding up the forces in the direction parallel to the plane and using Newton's second law ( $F = ma$ ) we get the following system of linear equations:

$$692,96 - 173,24 - T = 100a$$

$$346,48 - 129,93 + T - R = 50a$$

$$138,59 - 51,97 + R = 20a$$

or in matrix form::

$$\begin{bmatrix} 100 & 1 & 0 \\ 50 & -1 & 1 \\ 20 & 0 & -1 \end{bmatrix} \begin{bmatrix} a \\ T \\ R \end{bmatrix} = \begin{bmatrix} 519,72 \\ 216,55 \\ 86,62 \end{bmatrix}$$

To solve the system of linear equations, the matrix formula  $AX = B$  or  $X = A^{-1} * B$  is used. By using the help of Matlab obtained the following values:

$$a = 4,8045 \text{ m/detik}^2$$

$$T = 35.6671 \text{ N}$$

$$R = 10,1906 \text{ N}$$

The Matlab program is as follows:

```
>> a=[100 1 0; 50 -1 1; 20 0 -1]
```

```
a =
```

```
100    1    0
 50   -1    1
 20    0   -1
```

```
>> b=[519.72 ; 216.55 ; 86.62]
```

```
b =
```

```
519.7200
216.5500
 86.6200
```

```
>> x=inv(a)
```

```
x =
```

```
0.0059    0.0059    0.0059
0.4118   -0.5882   -0.5882
0.1176    0.1176   -0.8824
```

```
>> c=x*b c =
    4.8405
   35.6671
   10.1906
```

Where : a = 4.8045 m/sec<sup>2</sup>

T = 35.6671 N

R = 10.1906 N

While the second problem encountered in research related to the field of electrical engineering is electrical circuits. It includes the determination of current and voltage which is solved using Kirchoff's Law and Ohm's Law. Kirchhoff's law states that the algebraic sum of all currents entering a node must be zero, or  $\sum i_k = 0$  where all currents entering a node are considered positive. While Ohm's law states that the current through a resistor is related to the change in voltage and the electric current from the resistance is:

$$i_{ij} = \frac{V_i - V_j}{R_{ij}}$$

In this section will be presented about the problem of electrical circuits as follows:

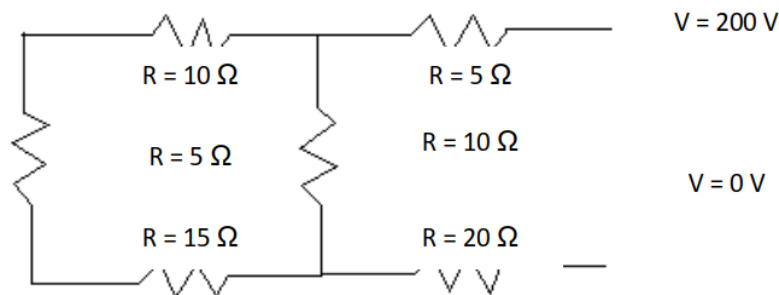


Figure 5. Electrical circuit problems

To solve the electrical circuit problem in Figure 5, the following steps are applied:

1. Label and symbolize all unknown good quantities. Determine the direction of the current in each part of the circuit. If the estimated current direction is reversed, the result will be negative but the magnitude is the same.
2. Set up a system of linear equations based on Kirchoff's Law and Ohm's Law.
3. Make a system of linear equations in the matrix form  $[A] [X] = [C]$
4. Determine the value of the matrix  $[X]$  by using the matrix formula  $[X] = [A]^{-1} [C]$



Based on these steps, the first step is to provide labels, symbols and current directions for each part in the circuit so that the problem in Figure 5 becomes Figure 6 as follows:

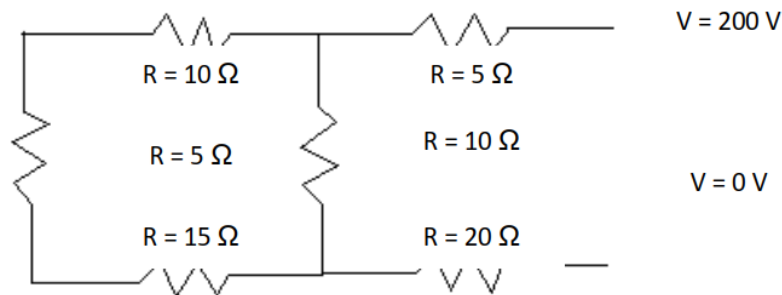


Figure 6. Assumed current direction

The second step is to construct a system of linear equations based on Kirchoff's Law and Ohm's Law. By using these two laws, there are four vertex equations and six stress equations.

Based on four node equations and six stress equations, ten systems of linear equations can be arranged as follows:

$$i_{12} + i_{52} + i_{32} = 0 \quad (7)$$

$$-i_{54} - i_{65} - i_{52} = 0 \quad (8)$$

$$i_{43} - i_{32} = 0 \quad (9)$$

$$i_{54} - i_{43} = 0 \quad (10)$$

$$10 i_{32} + V_2 - V_3 = 0 \quad (11)$$

$$5 i_{12} + V_2 = 200 \quad (12)$$

$$15 i_{43} + V_3 - V_4 = 0 \quad (13)$$

$$5 i_{54} + V_4 - V_5 = 0 \quad (14)$$

$$20 i_{65} + V_5 - 0 = 0 \quad (15)$$

$$10 i_{52} + V_2 - V_5 = 0 \quad (16)$$

The third step is to arrange the system of linear equations into a matrix form  $[A] [X] = [C]$ . The way is that each variable coefficient of current and electric potential difference is written as a representative of the matrix  $[A]$ . This writing is valid from equation (7) to (16). Example: in equation (7) the variables whose coefficient values are not equal to zero are variables  $i_{12}$ ,  $i_{32}$ , and  $i_{52}$  so that the  $i_{12}$ ,  $i_{32}$ , and  $i_{52}$  columns have a value of one each. While the coefficient of the other variables is zero. Thus the entire system of linear equations above becomes:

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 5 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 15 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 20 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 10 & 1 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} i_{12} \\ I_{32} \\ I_{43} \\ I_{54} \\ I_{65} \\ I_{52} \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 200 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

The fourth step is to calculate the value of matrix [X] by multiplying the inverse matrix [A] with matrix [B] with the help of Matlab software. Matrix [X] contains values i12, i32, i43, i54, i65, i52, V2, V3, V4, and V5. Thus, to determine the value of row elements of matrix [B] can be written:

$$\begin{pmatrix} i_{12} \\ I_{32} \\ I_{43} \\ I_{54} \\ I_{65} \\ I_{52} \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 5 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 15 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 20 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 10 & 1 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 200 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

```
>> A=[1 1 0 0 0 1 0 0 0 0;
0 0 0 -1 1 -1 0 0 0 0;
```

```

0   -1   1   0   0   0   0   0   0   0;
0    0  -1   1   0   0   0   0   0   0;
0   10   0   0   0   0   1  -1   0   0;
5    0   0   0   0   0   1   0   0   0;
0    0   5   0   0   0   0   1  -1   0;
0    0   0  15   0   0   0   0   1  -1;
0    0   0   0  20   0   0   0   0   1;
0    0   0   0   0  10   1   0   0  -1]

```

A =

```

1  1  0  0  0  1  0  0  0  0
0  0  0 -1  1 -1  0  0  0  0
0 -1  1  0  0  0  0  0  0  0
0  0 -1  1  0  0  0  0  0  0
0 10  0  0  0  0  1 -1  0  0
5  0  0  0  0  0  1  0  0  0
0  0  5  0  0  0  0  1 -1  0
0  0  0 15  0  0  0  0  1 -1
0  0  0  0 20  0  0  0  0  1
0  0  0  0  0 10  1  0  0 -1

```

```
>> B=[0; 0; 0; 0; 0; 200; 0; 0; 0; 0]
```

B =

```

0
0
0
0
0
200
0
0
0
0

```

```
>> C= inv(A1)
```

C =

0.8462	0.6154	0.7692	0.7308	-0.0077	0.0308	-0.0077	-0.0077	-0.0308	-0.0231
0.0385	-0.1538	-0.6923	-0.5577	0.0269	-0.0077	0.0269	0.0269	0.0077	-0.0192
0.0385	-0.1538	0.3077	-0.5577	0.0269	-0.0077	0.0269	0.0269	0.0077	-0.0192
0.0385	-0.1538	0.3077	0.4423	0.0269	-0.0077	0.0269	0.0269	0.0077	-0.0192
0.1538	0.3846	0.2308	0.2692	0.0077	-0.0308	0.0077	0.0077	0.0308	0.0231
0.1154	-0.4615	-0.0769	-0.1731	-0.0192	-0.0231	-0.0192	-0.0192	0.0231	0.0423
-4.2308	-3.0769	-3.8462	-3.6538	0.0385	0.8462	0.0385	0.0385	0.1538	0.1154
-3.8462	-4.6154	-10.7692	-9.2308	-0.6923	0.7692	0.3077	0.3077	0.2308	-0.0769
-3.6538	-5.3846	-9.2308	-12.0192	-0.5577	0.7308	-0.5577	0.4423	0.2692	-0.1731
-3.0769	-7.6923	-4.6154	-5.3846	-0.1538	0.6154	-0.1538	-0.1538	0.3846	-0.4615

>> X= C\*B

X =

6.1538  
-1.5385  
-1.5385  
-1.5385  
-6.1538  
-4.6154  
169.2308  
153.8462  
146.1538  
123.0769

>> V5= 123.0769, V4= 146.1538, V3= 153.8462, V2 = 169.2308, i52 = -4.6154, i65= -6.1538 i54 = -1.5385

After calculating with the help of Matlab software, the following results are obtained:

i12 = 6.1538 amperes; i32 = -1.5385 amperes; i43 = -1.5385 amperes;  
i54 = -1.5385 amperes; i65 = -6.154 amperes; i52 = -4.6154 amperes;  
V2 = 169.2308 volts; V3 = 153.8462 volts; V4 = 146.1538 volts;  
V5 = 123.0769 volts.

The negative sign obtained from the calculation results indicates that the direction of the example is opposite to the original direction. Thus the solution obtained from the electrical circuit in Figure 5 for more details is described below.

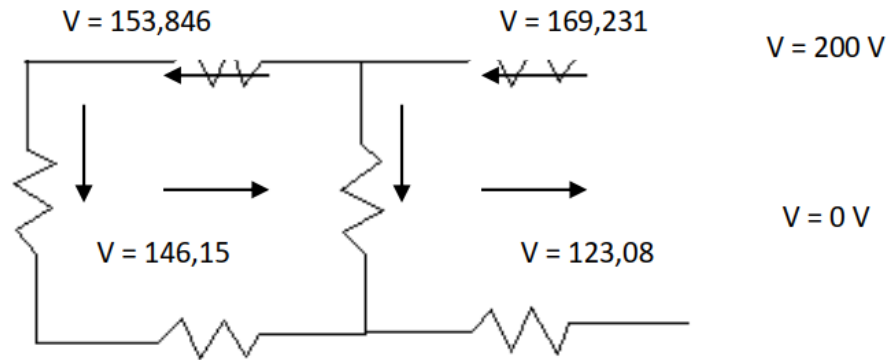


Figure 7. Solution of a system of linear equations of an electric circuit

Based on these results, it can be concluded that the system of linear equations with the form  $AX = B$  can be solved with the help of Matlab. Furthermore, the results of the research are the answers to the research problems.

#### 4 CONCLUSION

Based on the results of the research that has been done, the following conclusions can be written in the following sentences. To solve the problem in this study, the following steps were taken: (1) Formulating the problem into a system of linear equations (2) Writing down the results of linear equations in the form of a matrix  $AX = B$  (3) Writing Calculating the value of the X matrix as a representation of variables to be calculated (4) Calculate the value of the X matrix using the help of Matlab software, (5) The results of the calculation of the X matrix as a solution to the value of the research variable that has been determined.

This research can be continued to solve other problems that are still related to research on the  $AX = B$  matrix contained within the engineering field and outside the engineering field.

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