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POSITIONING ESTIMATION OF AUTONOMOUS CAR USING EXTENDED KALMAN FILTER

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ABSTRACT

In this paper, the Extended Kalman Filter (EKF) is proposed to estimate the position of an autonomous car. The EKF is chosen since it is the widely used estimation algorithm for nonlinear systems. The design of EKF is relatively easy compared to other versions of the KF modification. The used mathematical model in this simulation is kinematics in terms of nonlinear system. Based on the simulation results, the EKF can estimate the position of an autonomous car with comparatively small errors between the actual and the estimate positions. In this case, the reported RMSE for x-position and y-position is 0.1733 m and 0.1437 m, respectively. According to those results, the EKF method is applicable for the positioning estimation of autonomous cars.

KEYWORDS: *Autonomous Car, Nonlinear System, Positioning Estimation, EKF*

1 INTRODUCTION

The research and development of autonomous car have increased significantly in recent decades (Franco & Santos, 2019; Gao et al., 2010; Xu & Yuan, 2016). This is due to several underlying factors, namely its usefulness. Autonomous car is a car that has the ability to drive like controlled by a human. In developed countries this vehicle is increasingly being developed by several automotive companies, such as Tesla, Nissan, Toyota, Mercedes Benz, Volvo and others. Autonomous cars are also equipped with special features, such as digital camera, Global Positioning System (GPS,) radar, and automatic functions which drive the steering wheel to control acceleration and deceleration on the gas pedal. It is noted that the design of an autonomous car system is so complex, since it needs several engineering aspects.

In engineering, a branch dealing with the system design for controlling the movement of a vehicle is guidance, navigation, and control (Grewal et al., 2007). The discussed problem in this study is navigation. The aim of navigation is to ascertain the current position of a vehicle, so it can be monitored and controlled. Actually, the position of an autonomous vehicle can be determined by GPS. However, due to certain problems for example the effect of signals from the surrounding environment caused of obstacles including buildings, trees, and others, the accuracy of GPS is not perfect. In addition, the cracker attacks and others are also a threat to obtain the position information of a vehicle via GPS. In fact, the problem can be overcome by using an accelerometer. This tool can measure the acceleration of a vehicle, so the position of a vehicle can be known. Unfortunately, the two mentioned ways are often affected by errors, so it is better to combine the measurement and the estimation method.

The familiar estimation method often implemented to estimate the states and parameters is Kalman Filter (KF). The design of KF is based on the linear systems (Kim & Hyochoong, 2018), but the real problems that we encounter are related to the nonlinear systems. By this disadvantage, the KF is not suitable to handle those problems. One of the KF modifications which can be utilized to obtain the better estimation results in nonlinear systems is the Extended Kalman Filter (EKF). In general, the design of EKF is similar to KF, only that the state transition and observation matrices are obtained by using the Jacobians of the original system at the current estimates. The EKF is the widely used estimation algorithm for nonlinear systems, since its design is easier than other types, namely the Ensemble Kalman Filter (EnKF) and Unscented Kalman Filter (UKF). The contribution of this study is to apply the EKF method to estimate the autonomous car positions.

The structure of this paper is organized as follows. Section 2 explains the materials and methods including the mathematical model of autonomous car and the Extended Kalman Filter estimation method. Then, the simulation results are discussed in Section 3. Finally, the conclusion is presented in the last section.

2 MATERIALS AND METHODS

This section contains mathematical model of autonomous car and estimation method used to estimate the position of an autonomous car under measurement and process noise. The employed estimation method in this study is the Extended Kalman Filter.

2.1 Mathematical Model of Autonomous Car

To design estimation algorithm for an autonomous vehicle is easier if the mathematical model is known. Most of estimation methods are based on the mathematical model both in terms of linear and nonlinear. The model used in this paper is the nonlinear kinematics which can be modelled by the following equations of motion (Franco & Santos, 2019; Xi & Baras, 2007; Xu & Yuan, 2016):

$$\begin{cases} \dot{x} = v\cos\psi \\ \dot{y} = v\sin\psi \\ \dot{\psi} = \dfrac{v}{L}\tan\delta \\ \dot{v} = 0.5 \cdot T_h \end{cases} \tag{1}$$

where L is the autonomous car length, (x, y) is the position of an autonomous car as the global coordinates measured from rear wheel, v is the linear velocity of an autonomous car, ψ is the heading angle of an autonomous car w.r.t the x-axis, T_h is the throttle (positive if accelerated or negative if decelerated), δ is the steering angle where its value is positive when counter clockwise. The model can be simplified by assuming that adjusting the car can be done by the front wheel. In order to employ the estimation method, the model (1) can be written into state space model as follows:

$$\dot{\chi}(t) = \boldsymbol{f}\big(\boldsymbol{\chi}(t), \boldsymbol{u}(t)\big) \tag{2}$$

where $\boldsymbol{\chi} = [\,x \quad y \quad \psi \quad v\,]^T \in \mathbb{R}^4$ is the state variables and $\boldsymbol{u} = [\,\delta \quad T_h]^T \in \mathbb{R}^2$ is the manipulated or input variables. The system (2) is in continuous time, however the estimation methods such as Kalman Filter (KF) and its modification use the discrete system. To obtain the discrete nonlinear system, the Euler discretization can be utilized. By using Euler with the specific sampling time T_s, the system (2) can be rewritten in terms of discrete system as follows (Purnawan et al., 2021):

$$\boldsymbol{\chi}_{k+1} = \boldsymbol{\chi}_k + T_s \cdot \boldsymbol{f}(\boldsymbol{\chi}_k, \boldsymbol{u}_k) \tag{3}$$

where $k = \frac{t}{T_s}$. The sampling time should be chosen such that the characteristics of system (2) can be maintained.

2.2 Extended Kalman Filter

The Extended Kalman Filter (EKF) is the nonlinear estimation version of KF. In EKF, the state and measurement equations with additive noise can be described by the following equations (Ulusoy, 2022):

$$\begin{aligned} \boldsymbol{\chi}_k &= \boldsymbol{f}_d(\boldsymbol{\chi}_{k-1}, \boldsymbol{u}_k) + \boldsymbol{w}_k \\ \boldsymbol{z}_k &= \boldsymbol{g}(\boldsymbol{\chi}_k) + \boldsymbol{v}_k \end{aligned} \tag{4}$$

where $\boldsymbol{f}_d(\boldsymbol{\chi}_{k-1}, \boldsymbol{u}_k) = \boldsymbol{\chi}_{k-1} + T_s \cdot \boldsymbol{f}(\boldsymbol{\chi}_{k-1}, \boldsymbol{u}_k)$ which refers to Eq. (3) and $\boldsymbol{g}(\boldsymbol{\chi}_k)$ are the nonlinear differentiable functions, $\boldsymbol{z}_k \in \mathbb{R}^{n_z}$ is the observation variable at time k, the process and measurement noises can be symbolized as $\boldsymbol{w}_k \sim \mathcal{N}(0, \boldsymbol{Q}_k)$ and $\boldsymbol{v}_k \sim \mathcal{N}(0, \boldsymbol{R}_k)$ in which $\boldsymbol{w}_k$ and $\boldsymbol{v}_k$ are assumed to have Gaussian or normal distribution. The stages of EKF consist of the prediction and correction, so with these stages, EKF can result the good estimation. The algorithm of EKF can be summarized as follows (Herlambang, 2019; Liu et al., 2020):

Table 1. The algorithm of EKF

1. Initialization:	
$\widehat{\boldsymbol{\chi}}_0 = \overline{\boldsymbol{\chi}}_0$ $\boldsymbol{P}_0 = \boldsymbol{P}_{\chi_0}$	
2. Time update:	
Predicted state estimate:	$\widehat{\boldsymbol{\chi}}_k^- = \boldsymbol{f}_d(\widehat{\boldsymbol{\chi}}_{k-1}, \boldsymbol{u}_k)$
Predicted covariance estimate:	$\boldsymbol{P}_k^- = \boldsymbol{A}_k \boldsymbol{P}_{k-1} \boldsymbol{A}_k^T + \boldsymbol{Q}_k$
3. Measurement update:	
Kalman gain:	$\boldsymbol{K}_k = \boldsymbol{P}_k^- \boldsymbol{H}_k (\boldsymbol{H}_k \boldsymbol{P}_k^- \boldsymbol{H}_k^T + \boldsymbol{R}_k)^{-1}$

Updated state estimate:	$\hat{\chi}_k = \hat{\chi}_k^- + \boldsymbol{K}_k(\boldsymbol{z}_k - \boldsymbol{g}(\hat{\chi}_k^-))$
Updated covariance estimate:	$\boldsymbol{P}_k = (\boldsymbol{I} - \boldsymbol{K}_k\boldsymbol{H}_k)\boldsymbol{P}_k^-$

where the state transition and measurement matrices are defined by

$$\boldsymbol{A}_k = \left.\frac{\partial \boldsymbol{f}_d}{\partial \chi}\right|_{\hat{\chi}_{k-1},\boldsymbol{u}_k}$$

$$\boldsymbol{H}_k = \left.\frac{\partial \boldsymbol{g}}{\partial \chi}\right|_{\hat{\chi}_k^-}$$

Generally, the EKF is not the optimal estimator for nonlinear systems since both the state transition and the measurement matrices are linear. Additionally, the EKF may give the inappropriate results if the initial estimate and the process are incorrect. This problem can be caused by its linearization process.

3 SIMULATION RESULTS AND DISCUSSION

This section is to analyse the simulation results of the proposed estimation method for the chosen case. This simulation is performed in MATLAB R2021a on a computer with 8GB RAM and a core i5-1035G4. The initial conditions of state variables for this simulation are set to $\chi_0 = [-35, -75, 0, 20]^{\mathrm{T}}$, the sampling time is $T_s = 0.02\ s$, the initial state error covariance $\boldsymbol{P}_0 = 10^{-5} \cdot \boldsymbol{I}_4$ where $\boldsymbol{I}_4$ is the identity matrix with size 4×4, the process noise covariance is $\boldsymbol{Q}_k = \mathrm{diag}([0\ \ 0\ \ 10^{-3}\ \ 10^{-3}])$, and the measurement noise covariance is $\boldsymbol{R}_k = \mathrm{diag}([10^{-1}\ 10^{-1}])$. The final time used for this simulation is $t_{end} = 20\ s$. The simulation of the problem is carried out in Simulink MATLAB R2021a using control system toolbox. The block diagram design of EKF for autonomous car is depicted in the Figure 1.

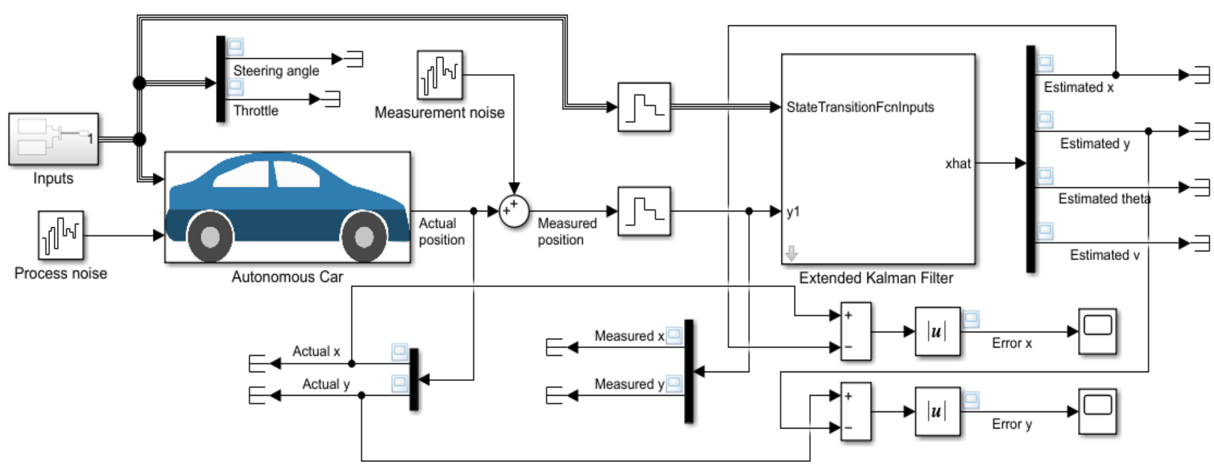

Figure 1: Block diagram of EKF for autonomous car system

Figure 1 is the modification version of EKF for pendulum system presented by (Ulusoy, 2022). The inputs of autonomous car system are generated using a signal builder obtained through NMPC simulation which is performed previously. The reference is constructed using driving scenario designer. According to Figure 1, the process noise is added to the inputs at each time of the simulation. The subsystem of autonomous car consists of the MATLAB functions constructed from Eq. (1).

The simulation of EKF for estimating the position of autonomous car compared to the actual position is shown in Figure 2.

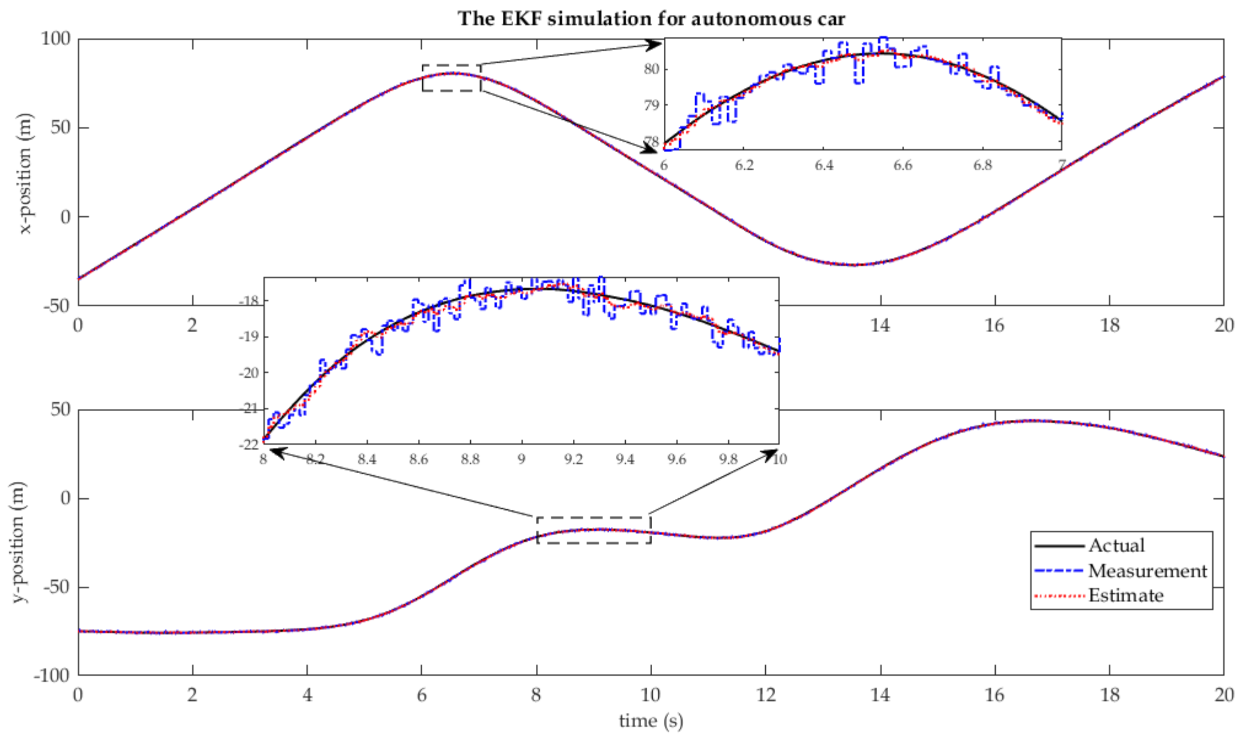

Figure 2: The position estimates of autonomous car using EKF

In Figure 2, the simulation is made for 20 seconds. Based on Figure 2, it can be observed that the estimate is close to the actual position both x-position and y-position. The measurement noise can affect the observed output, so the value can be far from the actual value. Figure 2 also shows that the EKF estimation method can produce the good estimate of autonomous car positions. To monitor the errors between the actual and estimate positions of the simulation, it is essential to show the errors in a figure. Figure 3 shows that the errors of the x-position and y-position are below 0.7 m in each time. To obtain the overall error of x-position and y-position, the Root Mean Square Error (RMSE) is calculated by the following equation:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{N}(\eta_i - \hat{\eta}_i)^2}{N}}$$

where η is the actual value, $\hat{\eta}$ is the estimate value, and N is the number of data. Based on the result of the errors depicted in Figure 3, the RMSE of x-position is about 0.1733 m and 0.1437 m for RMSE of y-position. According to those results, the RMSE values are relatively small, so it can be concluded that the EKF gives the good estimates.

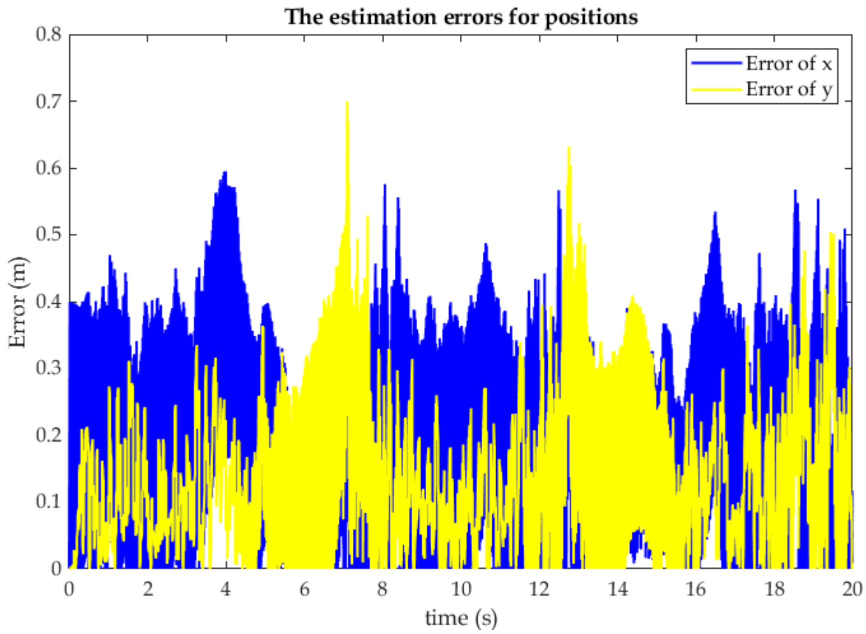

Figure 3: The errors between actual and estimate positions

In addition, the trajectory obtained from the estimation results compared to the actual value is shown in Figure 4.

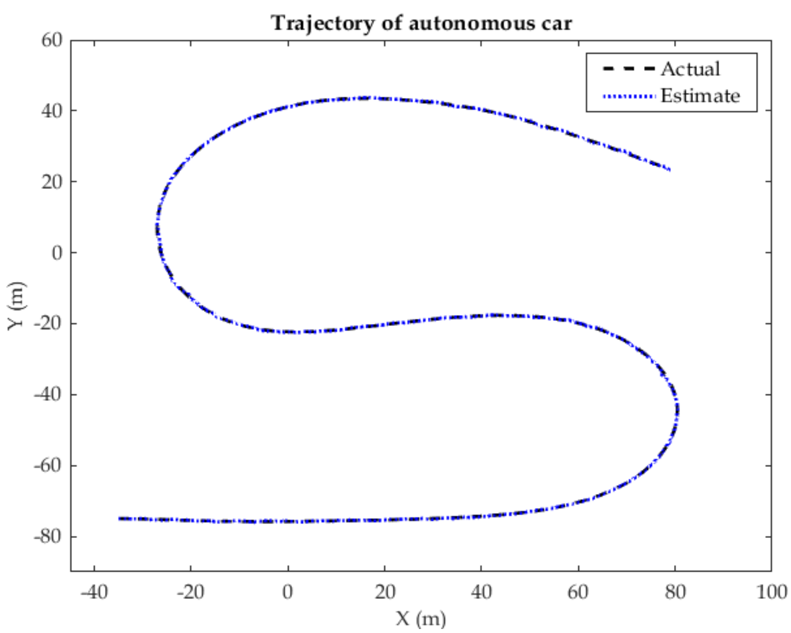

Figure 4: Trajectory of autonomous car

4 CONCLUSION

This paper employs the EKF estimation method to estimate the position of autonomous car under process and measurement noises. The design of EKF is constructed in Simulink in which the inputs for autonomous car system is obtained from NMPC. The collected inputs aim to make a trajectory of autonomous car. Based on the simulation results in previous section, the EKF gives the satisfactory estimates. The estimation errors in each time are small enough. Additionally, the RMSE of each position is also reported comparatively small. For further works, the EKF can be compared to other methods, particularly the UKF or even nonlinear Moving Horizon Estimation (MHE) to obtain the better estimates. Furthermore, the implementation of EKF will be more interesting if the method can be applied for the real problem of autonomous car.

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